A simple seismic imaging exercise

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After years of teaching courses on seismic techniques, from introductory to advanced, I have observed two things: (1) we learn best when we can visualize a concept, and (2) we learn by doing, not by observing what others do (Mike Graul taught me this).

I have let these two principles guide my development of this tutorial on imaging, which I have used many times in a course titled "Advances in seismic processing." In this exercise I use a simple dipping reflector to develop the concepts of NMO, prestack time migration, DMO, and poststack migration, and show how they are closely interrelated. The specific geometry was suggested to me years ago by Joong Chun and, although I never saw his development of the full exercise, I'm sure it would have gone something like this. By the time you get to the last figure, you will all say: "But that is so obvious!" What you won't appreciate is the blood, sweat, and tears I have put in to make it so obvious!

Now, let me give you a few guidelines for following this tutorial. First of all, don't just read it, but do it (guideline 2 above). All you need is squared graph paper, a pencil, a ruler, and a calculator. Second, if all of this is new to you, you may want to refer to other publications, such as Dave Hale's excellent *Dip Movement Processing* (SEG Course Notes Series, Volume 4). I have not developed the theory of imaging in this paper.

So, get out a clean notepad and let's start.

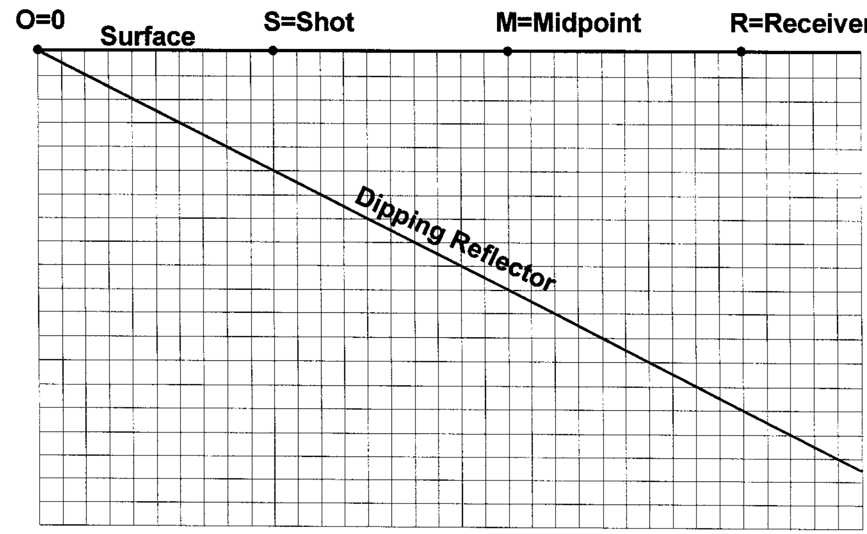

Figure 1. The basic geometry for the imaging exercise. This can be used as a template for the other exercises.

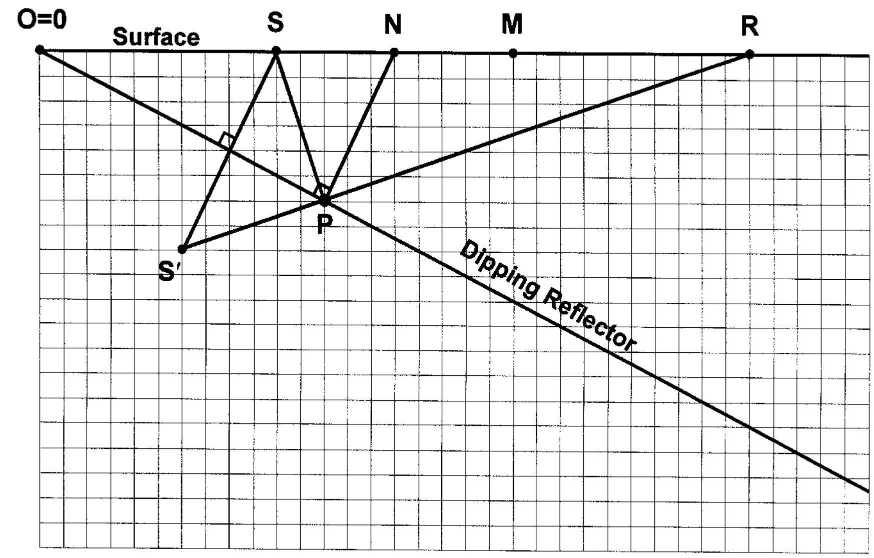

Figure 2. Construction of the true reflection travel path from *S* to *P* to *R*.

The basic geometry. In this exercise we are going to assume a single dipping bed within a constant velocity earth. Since the velocity (V) is constant, depth (d) and two-way seismic time (t) are simply scaled versions of each other, as expressed by the simple equation:

$$d = Vt/2, \qquad (1)$$

You can thus use the squares on your graph paper as the basic unit of time or depth, and their size is arbitrary. To make life really simple, we will do all of our measurements based on a unit square.

To start, turn the graph paper horizontally, and put a dot for the origin which is 5 squares down and 5 squares over to the right from the top left corner. Label this point O for origin, and note that it defines 0 for our initial work. Next, extend the surface horizontally along a straight line. Then, add a dipping reflector by connecting points that go to the right by 2 squares and vertically down by 1 square. This creates an angle of $\tan^{-1}(½) = 26.6°$ with respect to the horizontal (a strange angle but one that works well for this problem). Next, put a source (labeled S) 10 units to the right of the origin, and a receiver (labeled R) 20 units to the right of the source. This means that the offset, X, which is the distance between S and R, is 20 units. Finally, label the midpoint (halfway between S and R) M. Your initial geometry should look like Figure 1. You can use this initial geometry as a template for the rest of the exercises, so you may want to copy it several times.

We now want to draw the seismic reflection raypath. This would be easy if the reflector was flat but is a little trickier in the case of a dipping event. To create the raypath, we will use the image point technique. First, extend a line from the source S to the reflector at right angles to the reflector (simply reverse the initial dip, moving 1 square to the left and 2 squares down). This line should intersect the reflector at 4 squares below a point that is 2 squares to the left of the source. Then, extend the line the same distance on the other side of the reflector. This gives us the source image, which you can label S'.

Next, connect S' to the receiver R. This line will intersect the reflector at a point 6 units below a point on the surface that is 2 units to the right of S. This is the reflecting point, which you can label P. Connect S to P to get the downgoing part of the raypath. The upgoing raypath is from P to R. Finally, draw the raypath from reflecting point P to the surface, at right angles to the reflector. Label the point on the surface N, which is the normal to the reflecting point. All this is shown in Figure 2.

Notice that the total raypath length is given by the distance from S' to R, which you can see is equivalent to the combined lengths of SP and PR. By extending the points S' and P vertically to the surface, you can find the lengths of $S'R$, SP and PR, and thus prove that they are equivalent using the Pythagorean theorem. That is:

$$S'R = \sqrt{24^2 + 8^2} = \sqrt{640} = 8\sqrt{10} \approx 25.3$$

$$SP = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10} \approx 6.3$$

$$PR = \sqrt{18^2 + 6^2} = \sqrt{360} = 6\sqrt{10} \approx 19$$

The square roots are not only unavoidable, but they also are crucial to this problem, and it is important to remember them even when converting to an approximate value for the graph. The other raypath of interest is PN, the normal ray from the reflector. Notice that it intersects the surface at a point halfway between S and M (this is simply because of the dip angle and is not normally the case) and has a length of

$$PN = \sqrt{3^2 + 6^2} = \sqrt{45} \approx 6.7$$

NMO: The simplest imaging assumption. Now, let us assume that we don't know the subsurface geometry of the dipping bed we have just drawn (which, of course, is the normal situation), but we know that the reflection traveltime is $8\sqrt{10}$. The traditional NMO assumption is that the reflecting point is below the midpoint at a time given by the NMO equation:

$$t_r^2 = t_n^2 + X^2/V^2 \tag{2}$$

where $t_r = 2L/V$ = total reflection time; $t_n = 2d/V$ = NMO time; X = offset (= 20 units); and V = velocity.

The geometric interpretation of the NMO equation in a constant velocity earth is that the total raypath can be broken into two rays of equal length, L, and that there is a flat-lying reflector at a depth d below the surface. The velocity term (as well as the 2s) can be removed by multiplying through by $(V/2)^2$ which allows us to rewrite the NMO equation as:

$$L^2 = d^2 + h^2 \tag{3}$$

where $h = X/2$ = half-offset = 10 units; L = length of each raypath = $8\sqrt{10}/2 = 4\sqrt{10}$ units; and d = apparent depth to reflector.

Since we know the lengths of h and L, note that by rearranging the above equation, we can solve for the

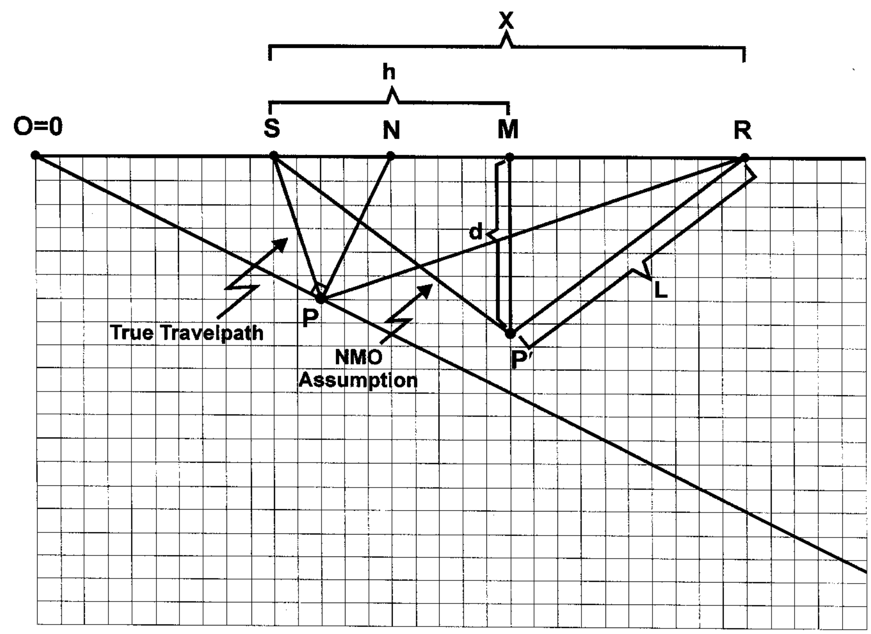

Figure 3. Comparison of the true reflection travel path with the zero dip NMO assumption, with the lengths from equation (3) annotated.

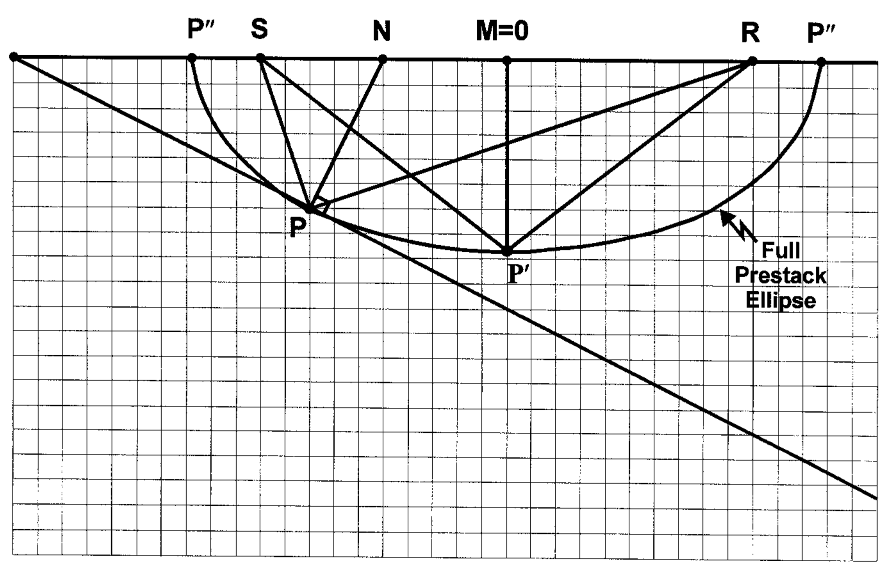

Figure 4. The full prestack time migration ellipse. Note that *P*, *P′* and *P″* are all valid points on the ellipse.

apparent depth to this mythical zero dipping reflector:

$$d = \sqrt{160 - 100} = \sqrt{60} \approx 7.75$$

Use the approximate value of 7.75 to place the apparent reflector vertically below the midpoint, but keep in mind that the true value is $\sqrt{60}$. Label the reflector as P'. This is shown in Figure 3, along with the definitions of X, L, h, and d.

The prestack time ellipse. We have now identified two points that could be the reflector, one using the true dipping geometry, and the other using the NMO assumption of zero dip. Actually, there are an infinite number of possible reflectors between dips of 0° and 90°. An obvious third point is symmetric with the true reflection point on the other side of the midpoint. (That is, for a dip from the right to the left of 26.6°). Finally, although a vertical reflector (dip = 90°) is geologically implausible, it will help us complete the picture. Such vertical reflectors would be at a distance of $4\sqrt{10}$ (approximately 12.65 units) from the midpoint in both directions. To visualize this, notice that a horizontal travel path would cover the distance from the source to the reflector on the left side of the source (or the right side of the receiver) twice, but the distance between the source and receiver only once. Label these points on your figure as P″. We have now defined five potential reflecting points, all with a travel path equal to 2L. These points all fall on an ellipse, given by the equation:

$$\frac{X^2}{L^2} + \frac{Z^2}{d^2} = 1 \qquad (4)$$

where: $L^2 = d^2 + h^2$ (from equation 3).

Equation (4) is referred to as the prestack time migration ellipse, and connects all the possible reflectors with dips between 0° and 90° from which our reflection could have been generated. To kinematically time migrate our data, we simply apply the following sequence of steps (there are also amplitude terms which are not discussed here):

NMO correction → spread over prestack ellipse → stack

The geometrical interpretation of the migration ellipse is that the halfwidth-width of the horizontal axis is L, and the half-width of the vertical axis is d (of course, half of the full ellipse is above the surface). You can sketch the full ellipse on your figure using the five points we have defined, and computing the rest of the X and Z coordinates from the following rearrangement of the ellipse equation (4):

$$Z = \pm d\sqrt{1 - (X/L)^2} \qquad (5)$$

where $d = \sqrt{60}$, and $L = 4\sqrt{10}$.

Note that you must redefine the origin to be at the midpoint, with negative distance to the left and positive distance to the right. Your completed figure should look like Figure 4. Table 1 shows the computed values.

Poststack migration. The traditional poststack migration approach can be written as:

NMO correction → stack → zero-offset migration.

In zero-offset migration, we can still use equation (4), but with an offset of 0. Thus, the ellipse simplifies to a circle of radius d. That is:

$$X^2 + Z^2 = d^2 \qquad (6)$$

Equation (6) can be thought of as zero-offset migration in a constant velocity earth. This circle can be sketched very easily on the figure we have been creating, and the result is shown in Figure 5. The computed values are shown in Table 2. The migration operator is simply a circle

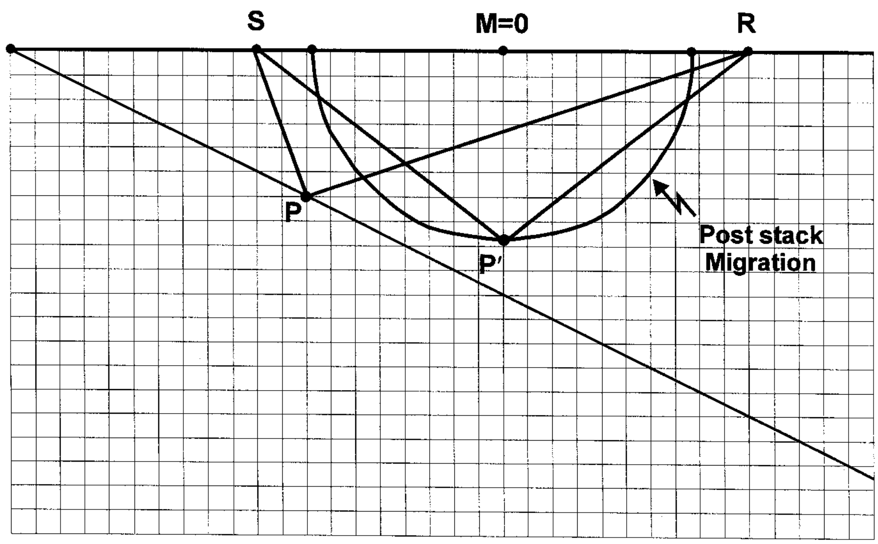

Figure 5. Poststack migration applied to the NMO corrected point P'. Note that the true reflection point P is never imaged.

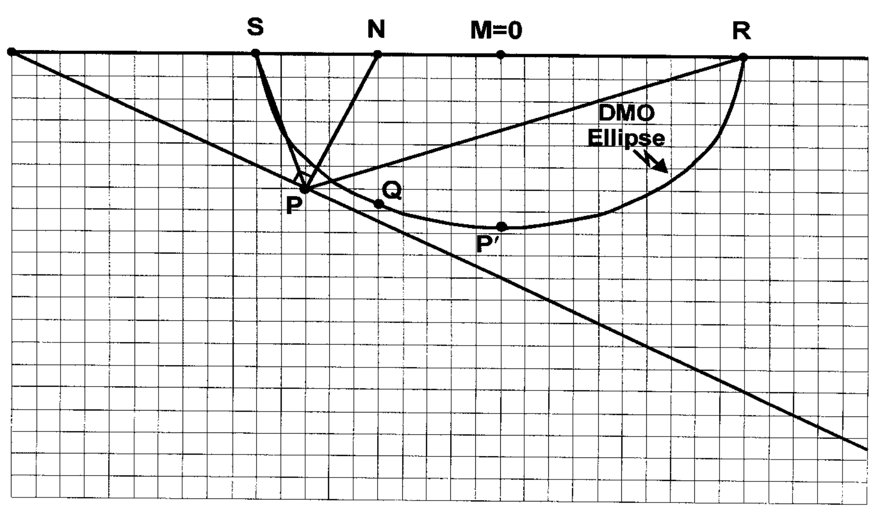

Figure 6. The full DMO ellipse. Notice that the true reflecting point is still not imaged. However, note that $NP = \sqrt{45}$ and, from Table 3, NQ is also equal to $\sqrt{45}$.

Table 1. *X*-*Z* pairs for the full prestack migration ellipse

X	*Z*
±12.65 = $4\sqrt{10}$	0.00
±12.00	2.45
±11.00	3.82
±10.00	4.74
±9.00	5.44
±8.00	6.00
±7.00	6.45
±6.00	6.82
±5.00	7.12
±4.00	7.35
±3.00	7.52
±2.00	7.65
±1.00	7.72
±0.00	7.75 = $\sqrt{60}$

Table 2. *X*-*Z* pairs for the post-migration circle

X	*Z*
±7.75 = $\sqrt{60}$	0.00
±7.00	3.32
±6.00	4.90
±5.00	5.92
±4.00	6.63
±3.00	7.14
±2.00	7.48
±1.00	7.68
±0.00	7.75 = $\sqrt{60}$

Table 3. *X*-*Z* pairs for the DMO ellipse

X	*Z*
±10.00	0.00
±9.00	3.38
±8.00	4.65
±7.00	5.53
±6.00	6.20
±5.00	6.71 = $\sqrt{45}$
±4.00	7.10
±3.00	7.39
±2.00	7.59
±1.00	7.71
±0.00	7.75 = $\sqrt{60}$

of radius $\sqrt{60}$. Notice, however, that this circle does not correctly image the reflector at any point. Thus, NMO followed by stack and poststack migration *is not correct for a dipping reflector*.

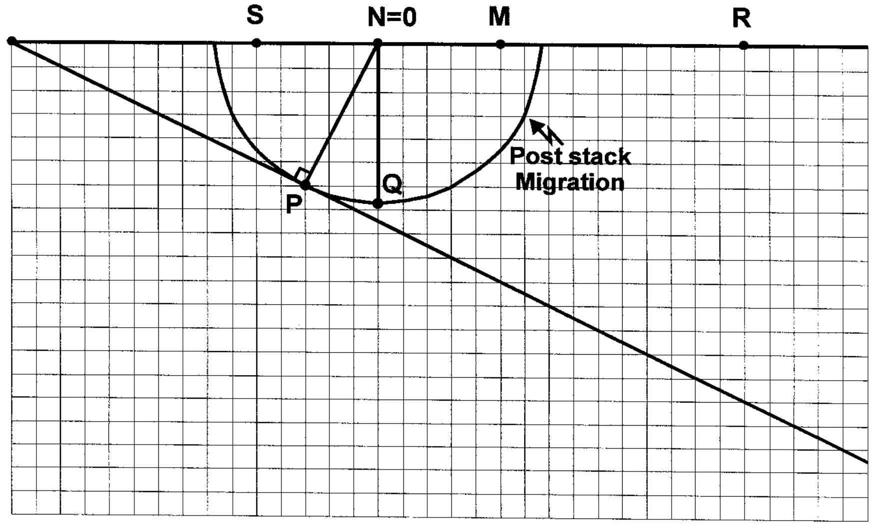

Figure 7. The poststack migration circle. Notice that point *Q* moves to point *P*.

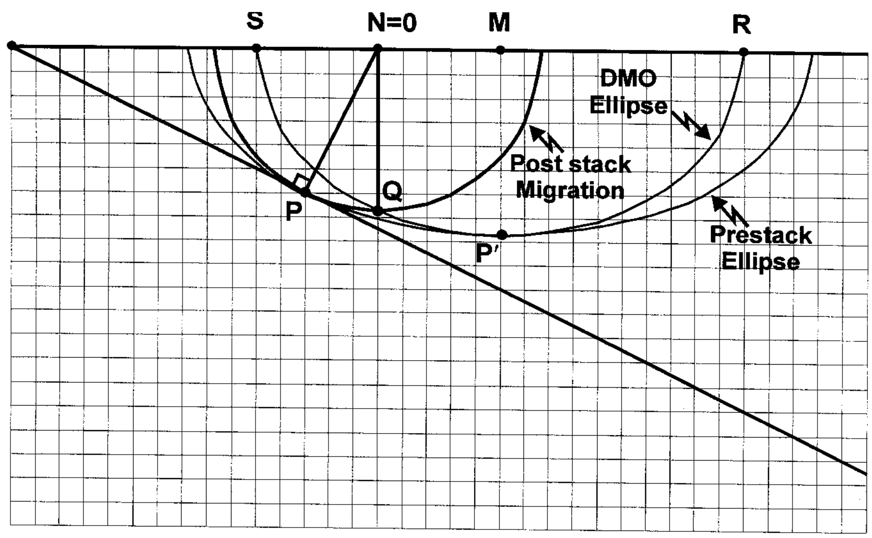

Figure 8. A final summary, showing the prestack ellipse, the DMO ellipse, and the poststack migration circle. Notice that *P′* moves to *Q*, and finally to *P*.

We have now looked at two approaches for imaging a reflector, NMO followed by full prestack time migration, which is correct for a dipping reflector and NMO followed by poststack time migration, which is incorrect for a dipping reflector. However, there is an intermediate step that we can insert between NMO and poststack migration to make it behave as prestack migration, and this step is called DMO, or dip moveout.

DMO: Partial prestack time migration. Although the derivation of the DMO operator is quite difficult, its interpretation is straightforward. Here it is in words:

The DMO operator is an ellipse passing through the NMO corrected value and the source and receiver coordinates.

Keeping in mind how we interpreted the full prestack ellipse, this leads to the following equation for the DMO ellipse:

$$\frac{X^2}{h^2} + \frac{Z^2}{d^2} = 1 \qquad (7)$$

In other words, we have simply changed the half-width of the ellipse on the horizontal axis from *L* to *h* but have kept the vertical half-width at *d*. To find the rest of the values of *X* and *Z* for this ellipse, equation (7) can be rewritten as:

$$Z = \pm d\sqrt{1-\left(\frac{X}{h}\right)^2} \qquad (8)$$

where $d = \sqrt{60}$ and $h = 10$.

Sketch the DMO ellipse on your graph paper and see how it relates to the other two curves. To help you, the DMO ellipse values are computed in Table 3, and the final ellipse is shown in Figure 6.

Let us focus on a single surface point, the one at *N*, which has an *X*-coordinate of -5. Remember, this is the normal projection of the raypath from the reflecting point *P*. This depth is easy to calculate from equation (8), giving:

$$Z = \sqrt{45} \approx 6.7$$

Recall that this was the length of normal raypath from the reflector! Plot this point below the *N* and label it *Q*. Can you see what is coming next?

Poststack migration after DMO. You will have noticed that the DMO ellipse still has not correctly repositioned our reflector. This is because we have not completed the sequence. For the final step, we will perform poststack migration. That is, draw a circle of radius $\sqrt{45}$ using point N as the center. Notice that this curve intersects the dipping reflector at the proper position! In other words, poststack migration will move points on the DMO ellipse to their correct reflection position (in our case, point *Q* has moved to point *P*). The values are shown in Table 4. The migration is shown in Figure 7.

Thus, we have shown graphically that DMO is indeed prestack partial migration, and that full prestack time migration can be achieved by :

NMO → DMO → stack → migration.

A final summary of the three curves — the full ellipse, the DMO ellipse, and the poststack migration circle — is shown in Figure 8.

Table 4. *X*-*Z* pairs for post-DMO migration

X	*Z*
±6.71 = $\sqrt{45}$	0.00
±6.00	3.00
±5.00	4.47
±4.00	5.39
±3.00	6.00
±2.00	6.40
±1.00	6.63
±0.00	6.71 = $\sqrt{45}$

Conclusion. In this simple graphical exercise, we have tied together three very important imaging concepts: NMO, DMO, and prestack time migration. The reason it all worked so well is that we assumed a constant velocity earth and a single dipping reflector. As the earth gets more structurally complex, these assumptions break down and we must introduce more advanced prestack time and depth migration concepts (see article by Ross in this issue). However, once the concepts of this exercise have been fully understood, you will have an easier time understanding these more complex ideas.

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