

## HOW THIN IS A THIN BED?†

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Based on reflective properties, a thin bed may be conveniently defined as one whose thickness is less than about  $\lambda_b/8$ , where  $\lambda_b$  is the (predominant) wavelength computed using the velocity of the bed. The amplitude of a reflection from a thin bed is to the first order of approximation equal to

$4\pi Ab/\lambda_b$ , where  $b$  is the thickness of the bed and  $A$  is the amplitude of the reflection if the bed were to be very thick. The equation shows that a bed as thin as 10 ft has, for typical frequency and velocity, considerably more reflective power than is usually attributed to it.

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### INTRODUCTION

For an unknown reason the geophysical industry appears to have grown up with some misconceptions on the reflective properties of so-called thin beds. What is the reflective behavior of thin beds? How thin must the bed be before the reflection has negligible amplitude? Our purpose is to consider such questions in elementary terms.

### GRAPHICAL ILLUSTRATION OF A TIME DERIVATIVE OF A WAVELET

Let us first examine the algebraic difference of two identical wavelets which are displaced slightly in time. Referring to Figure 1a, wavelets  $R_1$  and  $-R_2$  are identical except for the time difference  $\Delta T$  between them. Vertical lines between  $R_1$  and  $-R_2$  mark the difference in amplitude at successive simultaneous times, and this difference  $R_d$  is plotted in Figure 1b. The following properties of  $R_d$  are evident. (1) Wavelet  $R_d$  has zero amplitude in each half cycle at a time close to midway between the times when  $R_1$  and  $-R_2$  are at their maximum amplitude. That is, there is a 90-degree

phase shift between  $R_d$  and the mean of  $R_1$  and  $-R_2$ , the phase being advanced in time. (2) Correspondingly, whereas  $R_1$  exhibits an "M" form of character (when considering strongest peaks and trough),  $R_d$  has an "S" form of character (when considering strongest peak and trough). (3) The first maximum (a trough) of  $R_d$  arrives earlier than the first maximum (a trough) of  $R_1$ , and the last maximum (a peak) of  $R_d$  arrives later than the last maximum (a trough) of  $R_1$ . In total,  $R_d$  has a half cycle more than does  $R_1$ , and this incidentally is associated with a greater relative content of high frequency in  $R_d$  than in  $R_1$ .

The wavelet  $R_d$  is clearly the reflection from a thin bed, Figure 1c, when the acoustic impedance (product of velocity and density) in the medium above the bed is the same as that in the medium below the bed,  $R_1$  being the reflection from the top interface and  $R_2$  from the bottom interface (transmission loss and multiple reflections being neglected). The negative sign attached to  $R_2$  in Figure 1a, of course, accounts for the phase inversion at the bottom interface in this example. The time displacement  $\Delta T$  is equal to  $2b/V_b$  for vertical incidence, where  $b$  is the thickness of the bed and  $V_b$  is the velocity of the bed.

Since  $R_d$  is the difference between identical wavelets that are displaced in time,  $R_d$  approximates the time derivative of  $R_1$  when the time displacement is small. It is significant however that

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the *form* of the wavelet  $R_d$  still closely approximates the derivative of  $R_1$  even when  $b$  is as great as one-eighth of the wavelength  $\lambda_b$  computed using the velocity in the bed ( $\lambda_b = \tau V_b$ , where  $\tau$  is the predominant period of the wavelet). This is illustrated in the next figure to be discussed.

**EFFECT OF BED THICKNESS ON REFLECTION CHARACTER AND TIMING**

The traces in Figure 2d show reflections from a progressively thinner bed. As before, the velocity above the bed is the same as that below the bed, Figure 2a. The velocity of the bed itself is twice that of the superjacent and subjacent media. The wavelets  $R_1$  and  $R_2$  reflected from the upper and lower interfaces respectively, as well as the first-order multiple  $R_3$ , are shown in Figure 2c in terms

of the amplitude  $A_i$  of the incident wavelet  $R_i$ . The relations are for vertical incidence, and density changes are neglected. The first-order multiple reflection is so weak that for our present purposes it could also have been neglected, as are the higher order multiple reflections. The traces in Figure 2d were derived arithmetically by compositing  $R_1$ ,  $R_2$ , and  $R_3$  in a time relation corresponding to the respective bed thicknesses. The traces exhibit interplay between reflections from the top and bottom interfaces of the bed, producing destructive interference for  $b = \lambda_b/2$  and constructive interference especially for  $b = \lambda_b/4$ . Our attention however is to be directed toward the still thinner beds, where constructive interference is at first still active but with successively thinner

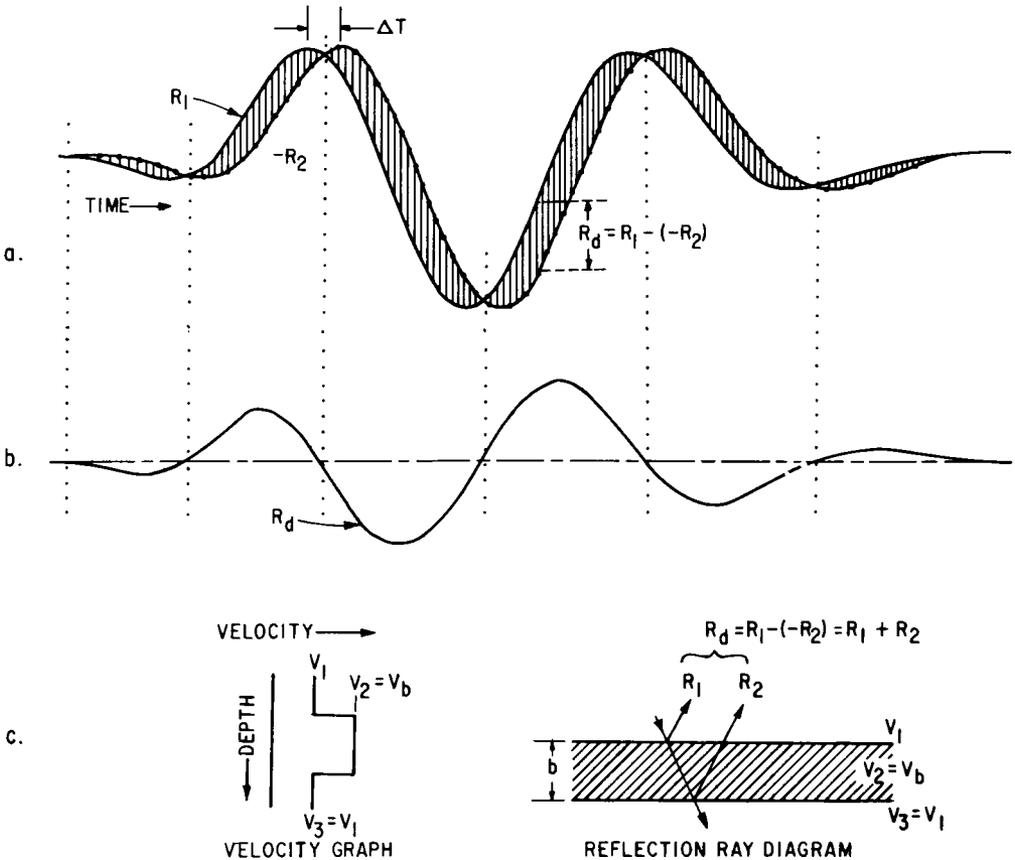


FIG. 1. The phase shift and change in character resulting from the difference of identical wavelets displaced slightly in time. a. Identical wavelets,  $R_1$  and  $-R_2$ , displaced by time  $\Delta T$ . b. Difference,  $R_d$ , between  $R_1$  and  $-R_2$ . c. Reflection from a thin bed in which  $V_3 = V_1$ .  $R_1$  and  $-R_2$  are identical except for a time displacement. (For simplicity, the transmission loss and multiple reflections are neglected and density is considered uniform.)

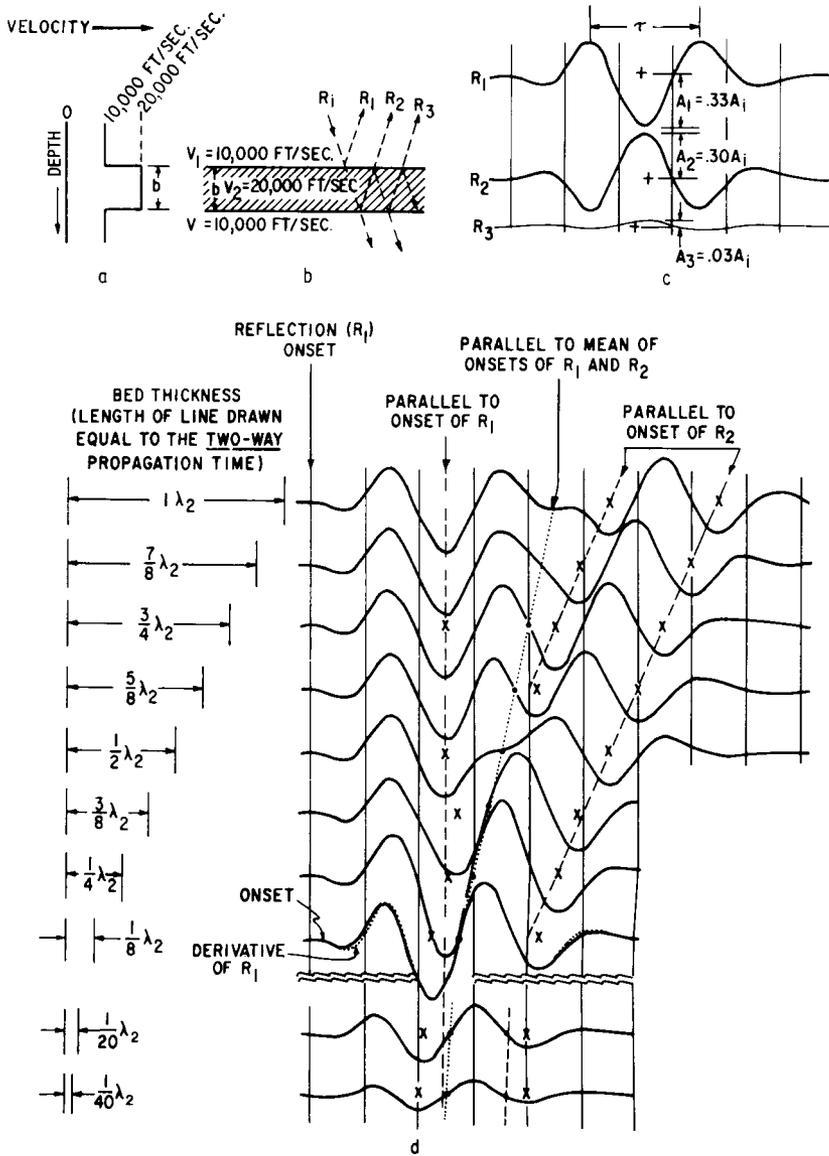


FIG. 2. Effect of bed thickness on the reflection. a. Velocity graph. b. Reflection ray diagram. c. Individual reflected waves that are composited using time delays computed from bed thickness. d. Form and relative timing of composite reflection as a function of bed thickness. X marks trough time. O marks zero-amplitude time ("center" of composite reflection). Timing line interval is  $0.5\tau$ .  $b$  = thickness of bed.  $\tau$  = predominant period of incident wavelet.  $\lambda_2$  = wavelength within bed ( $\lambda_2 = V_{2\tau}$ ). Amplitudes for composite reflections are all relative to the same incident wavelet  $R_i$ .

beds, destructive interference proceeds to extinguish the reflection.

When the bed is very thin, the character of the reflection is that of the time derivative of the incident wavelet and the timing is dictated by the time to the center of the bed. That substantially

the same character and timing exist for bed thickness as great as about  $\lambda_b/8$  is demonstrated on the trace for that bed thickness. The time derivative of the incident wavelet is shown there by the dotted-line wavelet, and we see that this almost duplicates the reflection on that trace. (In draw-

ing the time derivative wavelet, its amplitude was increased by a constant factor to match the amplitude of the reflection on the trace, and the onset of the time derivative wavelet was located at the mean of the onset times of  $R_1$  and  $R_2$ .) Thus, insofar as bed thickness alone is concerned, the character of the reflections is indistinguishable for beds whose thickness is less than about  $\lambda_b/8$ . For that reason it is appropriate to define a thin bed as one whose thickness is less than about  $\lambda_b/8$ . Two-way time through a thin bed would then be less than about  $\tau/4$ . A bed that is thin for one frequency is, of course, not necessarily thin for a higher frequency.

#### EFFECT OF BED THICKNESS ON REFLECTION AMPLITUDE

To the first order of approximation the central portion of wavelet  $R_1$  in Figure 1 may be treated as a sine wave whose maximum amplitude  $A$  is the mean between the amplitudes of the predominant peak and trough of  $R_1$ . This simplification permits an easy derivation of the approximate amplitude of reflection  $R_d$  from a thin bed. Referring zero time  $t_0$  to the mean of the deep-trough times of  $R_1$  and  $-R_2$ , the equations for the central portion of  $R_1$  and  $-R_2$  respectively are then

$$R_1 \cong -A \cos(t + b/V_b)2\pi/\tau, \quad (1)$$

and

$$-R_2 \cong -A \cos(t - b/V_b)2\pi/\tau, \quad (2)$$

where  $t$  is the time relative to  $t_0$  and  $\tau$  is the predominant period of the wavelet. By expanding the two equations and taking the difference, we obtain

$$R_d = R_1 + R_2 \cong [2A \sin 2\pi b/\tau V_b] \sin 2\pi t/\tau. \quad (3)$$

The term in brackets is approximately the maximum amplitude  $A_d$  of wavelet  $R_d$ . To the first order of approximation in the case of a thin bed,

$$\sin 2\pi b/\tau V_b \cong 2\pi b/\tau V_b.$$

So that

$$A_d \cong 4\pi A b/\tau V_b.$$

Since

$$\lambda_b = \tau V_b, \text{ we have}^1 \quad A_d \cong 4\pi A b/\lambda_b. \quad (4)$$

Therefore for thin beds the amplitude of the reflection is approximately proportional to the thickness of the bed and inversely proportional to the wavelength.

We note that reflections from beds that are generally considered very thin are not necessarily restricted to small amplitudes. For example, if  $b/\lambda_b = 1/20$ , we have  $A_d \cong 0.6A$ . That is, in a typical case of a reflection whose predominant frequency is 50 hz and a bed whose velocity is 10,000 ft/sec, the wavelength  $\lambda_b$  is 200 ft, so that a bed whose thickness is only 10 ft would still have about 0.6 of the amplitude that would result if the bed were very thick. If the bed were to be only 5 ft thick, the factor would still be fairly large, i.e., 0.3 instead of 0.6. These magnitudes may be seen in the bottom two traces respectively of Figure 2d, comparing them with the top trace.

The above conditions apply only to thin beds for which the two media bounding the bed have the same acoustic impedance. The relations do not apply when the two bounding media have appreciably different acoustic impedance, since in that case not only is a thin bed involved but also an acoustic change in the absence of the thin bed. It is then generally sufficiently accurate to consider that the reflection from the bed is a composite of the following two reflections: (1) the reference reflection, i.e., the reflection which would result in the absence of the thin bed, and (2) the time-derivative type of reflection associated with the thin bed itself, for which the acoustic impedance above and below the bed is the same and is equal to the acoustic impedance of the medium which the bed replaces. The effect of the thin bed may then be reckoned in terms of the relative strength and phase relation between the reference reflection and the time-derivative reflection. The relation may be shown readily by vectorial representation.

<sup>1</sup> The exact equation for reflection from a single imbedded layer, considering simple harmonic waves and accounting for transmission loss (but not absorption loss), is given in Rayleigh (1945). The equation, adapted to our notation, is

$$A_d = A(1+r)^2[(2r \cot 2\pi b/\lambda_b)^2 + (1+r^2)^2]^{-1/2},$$

where  $r$  is the ratio of acoustic impedances. The quantity  $A_d$  obtained from this equation differs from  $A_d$  in equation (4) by no more than only 12 percent when the bed is thin,  $b/\lambda_b < \frac{1}{8}$ , and when  $\frac{1}{2} < r < 2$ , the range of acoustic contrast usually encountered in practice.

## RESOLVING POWER

A definition of the term "thin bed" involves the concept of resolving power. Resolving power is the ability to distinguish between the properties of two (or more) elements. The elements that we have been considering here are the reflecting interfaces of a bed. Resolving power is illustrated in Figure 2 as follows. When bed thickness  $b$  is large enough that the individual reflected wavelets from each of the two interfaces are completely separated in time, the trace on the record, of course, potentially yields maximum possible information for each of the interfaces. As the bed thickness diminishes, more and more of the energy becomes a composite for the two reflections. That is, there is successively less data for each of the reflections separately but more data in the form of combination of the two reflections. This trend continues until the thickness is equal to about  $\lambda_b/8$ . For this and still thinner beds, substantially the only information left is for the combination of the two reflections and, therefore, substantially none for the individual reflections.<sup>2</sup> At that point,

<sup>2</sup> The fact that information only on a combination of reflections is available means that substantially the same reflection can be obtained for a very wide assortment of thin beds, which in themselves may consist of different laminas. It may therefore be of interest to state that to the first order of approximation the only conditions which these beds must fulfill to qualify in this assortment are the following: (1) the overall bed must be thin, (2) the total area between the time versus velocity curve and the reference velocity line must be the same for the overall bed, treating the respective areas of the bed algebraically, relative to the reference

therefore, resolving power may be said to be lost and the point may be loosely called the theoretical threshold of resolution. Practically, a number of other factors are involved that will determine the threshold of resolution. For example, with the presence of noise the broadening of the wavelet from  $b = \lambda_b/8$  to  $b = \lambda_b/4$  in Figure 2 may be obscured, thus forcing the threshold of resolution to the thicker bed. The threshold of resolution therefore depends not only on the predominant frequency of the incident wavelet but also on the signal-to-noise ratio. Still other factors include the form and duration of the incident wavelet, the degree to which this wavelet is known prior to the analysis, and the analytical tools used.

When the whole of a thin bed rather than the individual interfaces is to be considered, a different threshold of resolution is brought into play. For example, if either the velocity system or the bed thickness remains constant, the change of bed thickness or velocity, respectively, can be determined under favorable circumstances for a bed whose thickness is considerably less than  $\lambda_b/8$ . Measurements are then made on the change in reflection time and/or the change in reflection amplitude.

## REFERENCE

Rayleigh, John William Strutt, 1945, *The theory of sound*: New York, Dover Publishing Co., v. 2, p. 88.

velocity line; (3) the geometric center of this area must be at the same record time, again considering the respective areas algebraically relative to the reference velocity line.